

03 - Fractions

Numerical Reasoning Test Preparation Notes and Worked Examples

This document is part of a set of resources designed to help people prepare for numerical reasoning tests. However, they could also be used by anyone who wants to improve their maths skills, especially how to use a calculator to work with fractions, percentages, ratios and averages.

There are six topic areas covered in these documents (Calculator Skills; Numbers; Fractions; Percentages; Ratio and Proportion; and Averages). For each topic area there are two documents: one is a set of notes and worked examples, and the other is a question pack.

Each set of notes and each question pack is divided up into a number of skills. For each skill, read through the notes and examples and then try the questions for that skill from the question pack. I advise that you work through the skills in order, only moving onto the next one once you have mastered the current one (this is called *mastery learning*).

Enter a given fraction into a calculator.

Examples

- 1) Enter the fraction $\frac{17}{29}$ into your calculator.

Notes

There are two main ways to enter a fraction into a calculator.

The first, and perhaps most obvious, is to use the fraction button.

The label and location of the fraction button varies between calculators. On many calculators it has a symbol which looks like a fraction but with boxes instead of the numbers, and is found on the far left of the second row from the top (or the third row if you count the row with the SHIFT, ALPHA, MODE and ON buttons).

The way that the fraction button works also varies between calculators.

On some calculators, you start by pressing the fraction button, then you type in the top number (the numerator), then you press $\rightarrow$ or $\downarrow$, and then you type in the bottom number (the denominator).

To do Example 1 using the fraction button on a calculator that works like this, we would press the following:

- Fraction Button
- 17
- $\rightarrow$ or $\downarrow$
- 29

On other calculators, you start by typing the top number, then you press the fraction button, then you type the bottom number.

To do Example 1 using the fraction button on a calculator that works like this, we would press the following:

- 17
- Fraction button
- 29

As you can see, entering a fraction using the fraction button is a bit quicker on calculators that use the second approach.

For either type of calculator, if you want to continue your calculation after the fraction, you need to press $\rightarrow$ to exit the fraction. Otherwise, everything else you type will be inserted onto the bottom of the fraction.

For example, if we wanted to multiply $\frac{17}{29}$ by 120 using the fraction button on a calculator that uses the first approach, we would have to press:

- Fraction Button
- 17
- $\rightarrow$ or $\downarrow$
- 29
- $\rightarrow$
- x
- 120
- =

As you can see, using the fraction button to enter fractions can be quite time consuming due to the large number of button presses needed. This is made worse by the fact that the fraction button and the $\rightarrow$ and $\downarrow$ buttons are small buttons up at the top of the calculator, far away from the numbers and the = button.

Therefore, it may be quicker to use the second method of entering fractions, which is to use the division button.

Every fraction can be thought of as a division. Specifically, the fraction is equal to its top number divided by its bottom number. For example, $\frac{17}{29}$ is the same as $17 \div 29$. Therefore, to enter a fraction, we can simply type in the relevant division.

To do Example 1 using the division button, we simply press:

- 17
- $\div$
- 29

If we press =, the calculator will display the fraction: $\frac{17}{29}$.

One disadvantage of using the division button is that if we are using the fraction in a calculation then we may need to put brackets around the division. For example, if we wanted to divide 100 by $\frac{17}{29}$ we would have to type $100 \div (17 \div 29)$. If we didn't use brackets in this situation, the calculator would do the two divisions from left to right, which is not what we want.

We could avoid this need for brackets by doing the calculation in two steps. First we would type in $17 \div 29$ and press =. Then we would type $100 \div \text{Ans}$ and press =.

One advantage of using the division button is that we don't have to press $\rightarrow$ to exit the fraction. For example, above we used the fraction button to multiply $\frac{17}{29}$ by 120 and found that it took a lot of button presses (and a lot of moving our thumbs up and down the calculator). Using the division button, we would simply type $17 \div 29 \times 120$ and press =. There is no need for brackets because the calculator will do the division and multiplication from left to right, which is what we want.

This is only 10 button presses. Using the fraction button it takes 12 or 11 button presses, depending on whether the calculator uses the first or the second approach. And crucially, when we use the division button we are only using the big buttons at the bottom of the calculator.

Therefore, my recommendation to you would be that in most situations it is better to ignore the fraction button and just use the division button to enter fractions. However, it is a matter of personal preference and the most important thing is that you use a method you are comfortable with.

Find the simplest form of a given fraction.

Examples

- 1) Express the fraction $\frac{91}{63}$ in its simplest form.
- 2) What is the simplest form of the fraction $\frac{68}{153}$?

Notes

A scientific calculator will find the simplest form of a fraction for you. All you have to do is type in the fraction and then press =. You can type in the fraction using the fraction button or the division button.

To do Example 1, we simply type the fraction $\frac{91}{63}$ into the calculator (or type in $91 \div 63$) and hit =. The calculator will display the simplest form of the fraction, which in this case is $\frac{13}{9}$.

For Example 2, we simply type the fraction $\frac{68}{153}$ (or type in $68 \div 153$) and hit =. The calculator will show the simplest form of the fraction, which is $\frac{4}{9}$.

Add, subtract, multiply or divide two given fractions.

Examples

1) Add $\frac{5}{8}$ to $\frac{7}{4}$.

2) Subtract $\frac{3}{7}$ from $\frac{8}{9}$.

3) Multiply $\frac{3}{5}$ by $\frac{2}{11}$.

4) Divide $\frac{7}{12}$ by $\frac{9}{5}$.

Notes

To do these, we simply type the calculation into the calculator and press =. We can enter the fractions using either the fraction button or the division button.

To do Examples 1 to 4 using the fraction button we would input:

1) $\frac{5}{8} + \frac{7}{4} =$

Which gives $\frac{19}{8}$.

2) $\frac{8}{9} - \frac{3}{7} =$

Which gives $\frac{29}{63}$.

3) $\frac{3}{5} \times \frac{2}{11} =$

Which gives $\frac{6}{55}$.

4) $\frac{7}{12} \div \frac{9}{5} =$

Which gives $\frac{35}{108}$.

If we use the division button, we do not need brackets for adding, subtracting or multiplying two fractions, but we do need them for dividing one fraction by another.

For adding or subtracting fractions, we do not need brackets because the calculator will automatically do the two divisions before the addition or subtraction. Therefore, we can do examples 1 and 2 as follows:

$$1) 5 \div 8 + 7 \div 4 =$$

Which gives $\frac{19}{8}$.

$$2) 8 \div 9 - 3 \div 7 =$$

Which gives $\frac{29}{63}$.

For multiplying fractions, we might think that we need to use brackets because we want the calculator to do the two divisions first and then the multiplication. In Example 3, this would look like this:

$$(3 \div 5) \times (2 \div 11) =$$

Which gives $\frac{6}{55}$.

This is the correct answer, however, we don't actually need the brackets. We can do it more quickly without them, like so:

$$3) 3 \div 5 \times 2 \div 11 =$$

Which gives $\frac{6}{55}$.

When we don't include the brackets, the calculator works from left to right: first it divides 3 by 5 to give $\frac{3}{5}$, then it multiplies $\frac{3}{5}$ by 2 and divides the result by 11. This may seem wrong: surely after we get the $\frac{3}{5}$ we want to divide 2 by 11 to give $\frac{2}{11}$ and then multiply the $\frac{3}{5}$ by the $\frac{2}{11}$? However, multiplying by 2 and then dividing by 11 is actually the same thing as multiplying by $\frac{2}{11}$, so we get the same result regardless of whether or not we use the brackets.

I would argue that adding, subtracting and multiplying fractions using the division button is much quicker than doing these things using the fraction button.

For dividing fractions, we do need to use brackets with this method, otherwise the calculator would work from left to right, which would give the wrong answer. Therefore, we do Example 4 as follows:

4) $(7 \div 12) \div (9 \div 5) =$

Which gives $\frac{35}{108}$.

Even despite the need to use brackets, this is arguably still quicker and easier than using the fraction button because it is less fiddly to do on the calculator. This is, of course, personal preference. Perhaps try doing Example 4 both ways and see which you prefer.

Convert between fractions and decimals.

Examples

- 1) Convert $\frac{5}{8}$ to a decimal.
- 2) What is 0.4375 as a fraction?

Notes

A scientific calculator will convert between the fraction and decimal representations of a number for you. This exact way to do this varies from one model of calculator to the next. If the instructions below don't work for your calculator, make sure you find out how to do this on your calculator.

The calculator has a button which is labelled 'S $\leftrightarrow$ D' or something similar. This button is used to toggle between the fraction and decimal representations of a number.

To start, type the fraction or decimal that you are trying to convert into the calculator and press =. By default, most calculators will show you the fraction version. If you need the decimal, press S $\leftrightarrow$ D.

In Example 1, we start by typing $\frac{5}{8}$ into the calculator (either using the fraction button or by typing $5 \div 8$) and then press =. The output of the calculator now says $\frac{5}{8}$. We then press S $\leftrightarrow$ D and the calculator shows us the decimal representation of the number, which is 0.625.

In Example 2, we type in 0.4375 and press =. The calculator output shows $\frac{7}{16}$. This is 0.4375 as a fraction and is therefore the answer.

Convert between top heavy fractions and mixed numbers.

Examples

1) Write $2\frac{2}{7}$ as a top heavy fraction.

2) Write $\frac{14}{3}$ as a mixed number.

Notes

A top heavy fraction is a fraction in which the top number is bigger than the bottom number. For example, $\frac{17}{5}$ is a top heavy fraction because 17 is bigger than 5. A top heavy fraction has a value greater than 1 (for example, if we convert $\frac{17}{5}$ to a decimal we get 3.4).

Every top heavy fraction can also be written as a mixed number, which is a whole number with a fraction next to it. For example, the top heavy fraction $\frac{17}{5}$ can be written as the mixed number $3\frac{2}{5}$. This mixed number is pronounced, "*Three and two fifths*". A simple way to think of it is as 3 plus $\frac{2}{5}$, or as $\frac{2}{5}$ more than 3.

The reason that the top heavy fraction $\frac{17}{5}$ and the mixed number $3\frac{2}{5}$ are equal to each other is as follows:

$$\frac{17}{5} \text{ is equal to } \frac{15}{5} + \frac{2}{5}$$

...and...

$$\frac{15}{5} \text{ is equal to } 3 \text{ (because } 15 \div 5 = 3\text{)}$$

...therefore...

$$\frac{17}{5} \text{ is equal to } 3 + \frac{2}{5}, \text{ which is the same thing as } 3\frac{2}{5}.$$

Don't worry if that explanation didn't make sense to you. The important thing to know is that every top heavy fraction can be converted to a mixed number and every mixed number can be converted to a top heavy fraction. We can use our calculators to do the hard work for us.

In Example 1, we need to convert a mixed number to a top heavy fraction. To do this we need to enter the mixed number into the calculator.

On your calculator, there should be a way for you to enter a mixed number. On many calculators, you have to press SHIFT and then press the fraction button. You can then enter the mixed number (using the $\rightarrow$ button to move between the different parts of the mixed number).

So in Example 1, we would press the following buttons:

- SHIFT
- Fraction button
- 2
- $\rightarrow$
- 2
- $\rightarrow$
- 7

(The exact method may vary between different calculator models).

We can then press = and the calculator will automatically convert what we have entered to a top heavy fraction. The calculator should display $\frac{16}{7}$, which is the answer.

As you might have guessed, we don't actually have to use the fraction button for this. We could instead simply type in $2 + 2 \div 7$ and then press =.

Note: if your calculator displays $2\frac{2}{7}$ instead of $\frac{16}{7}$, then this means your calculator is set to automatically display mixed numbers instead of top heavy fractions. This is not a problem as the calculator provides a way to convert the output between the two, as explained below. Read the instructions below for Example 2 and then you will know what to do for Example 1. If you want to, you can change this setting so that your calculator automatically displays top heavy fractions. Depending on your calculator model, you may be able to find this setting by going into SETUP - otherwise, consult your calculator's user manual (you may be able to find a copy of it online).

In Example 2, we need to convert the top heavy fraction $\frac{14}{3}$ into a mixed number.

We start by entering the fraction $\frac{14}{3}$ into the calculator using our method of choice. We then press =. The calculator will display $\frac{14}{3}$ (unless your calculator is set to automatically display mixed numbers, in which case it will display the answer and your work is done).

The calculator provides a way to convert the output between top heavy fractions and mixed numbers. On many calculators, you do this by pressing SHIFT and then pressing the 'S $\leftrightarrow$ D' button.

If you do this once, the calculator will display $4\frac{2}{3}$, which is the answer.

Find a given fraction of a given amount.

Examples

1) What is $\frac{3}{7}$ of 1261?

2) What is $\frac{12}{5}$ of 72?

Notes

To find a given fraction of a given amount, simply multiply the fraction by the amount using your calculator.

To do Example 1, we just multiply $\frac{3}{7}$ by 1261.

We can enter the fraction $\frac{3}{7}$ into the calculator by using the fraction button or by typing in $3 \div 7$. If we are using the division button to input the fraction, we can just type in $3 \div 7 \times 1261$, without using brackets. Division and multiplication have equal priority in the order of operations, so the calculator will simply work from left to right in this situation.

Multiplying $\frac{3}{7}$ by 1261 gives 540.4285714, which is the answer.

To do Example 2, we multiply $\frac{12}{5}$ by 72 (by either typing in $\frac{12}{5} \times 72$ or typing in $12 \div 5 \times 72$, and then pressing =), which gives 172.8.

Note that 172.8 is bigger than 72. This is because $\frac{12}{5}$ is a top heavy fraction.

Calculate what fraction a given amount is of another given amount.**Examples**

- 1) What fraction of 567 is 49?
- 2) What fraction of 81 is 153?
- 3) What is 156 as a fraction of 276?

Notes

To find what fraction A is of B, we simply need to divide A by B on the calculator. The calculator will automatically give us the answer as a fraction in its simplest form (unless the answer is a whole number, in which case the answer is simply that number over 1).

To do Example 1, we simply type $49 \div 567$ into the calculator and press =. The calculator then outputs the answer: $\frac{7}{81}$.

When we do these questions, we need to read the question carefully to know which number to divide by which. In Example 1, the question asks, "What fraction **of** 567 is 49?". The number with the word '**of**' in front of it is the number that we need to divide by. Note that this will not always be the bigger number.

In Example 2, we are asked, "What fraction of 81 is 153?". Since we are instructed that it should be a fraction "of 81", we know that 81 is the number to divide by. Therefore, we type in $153 \div 81$ and press $=$. This gives us the answer: $\frac{17}{9}$.

Example 3 is worded a bit differently. But again, we look for the word "of". The question is asking for 156 as a fraction "of 276", so we need to divide by 276. We type in $156 \div 276$ and press $=$. This gives us the answer: $\frac{13}{23}$.

Given an amount and what fraction it is of another amount, calculate the other amount.

Examples

1) 34.8 is $\frac{5}{7}$ of another number. What is that number?

2) $\frac{9}{5}$ of a number is 540. What is that number?

Notes

There are two ways to do this type of question:

The first is to divide the number we are given by the fraction. To understand why this works, let's call the number we are looking for X. In Example 1, we know that 34.8 is $\frac{5}{7}$ of X (or in other words, $\frac{5}{7}$ times X). We can write this as an equation:

$$\frac{5}{7} \times X = 34.8$$

Dividing both sides by $\frac{5}{7}$ gives us:

$$X = 34.8 \div \frac{5}{7}$$

When carrying out this division, we can either enter $\frac{5}{7}$ using the fraction button or using the division button. If we use the division button, we need to use brackets to ensure that the two divisions are done in the right order, like so: $34.8 \div (5 \div 7)$. This gives the answer, which is 48.72.

The second method is to divide 34.8 by 5 and then multiply it by 7. To understand why this works, remember that 34.8 is $\frac{5}{7}$ of the answer. If we divide it by 5 then we get $\frac{1}{7}$ of the answer. If we then multiply this by 7 we get the answer. We can do this in one step on the calculator without using brackets. We simply have to type in $34.8 \div 5 \times 7$ and press $=$. Because division and multiplication are given equal priority in the order of operations, the calculator will work from left to right, which is the order that we want in this case.

To find the answer to Example 2, we can do any of the following:

- Type in $540 \div \frac{9}{5}$ and press =.
- Type in $540 \div (9 \div 5)$ and press =.
- Type in $540 \div 9 \times 5$ and press =.

Whichever method we use, we get 300. Note that this is smaller than 540 (the number we started with). This is because $\frac{9}{5}$ is a top heavy fraction.

Increase or decrease a given amount by a given fraction of itself.

Examples

- 1) The number 2000 is increased by $\frac{3}{8}$ of itself. What number does this give?
- 2) The number 57 is decreased by $\frac{7}{12}$ of itself. What number does this give?

Notes

There is more than one way to do this type of question, but we will just look at one method which requires a small number of button presses on the calculator.

First, we find out what the given fraction of the given amount is. So, in Example 1, we start by finding out what $\frac{3}{8}$ of 2000 is.

As previously explained, we can do this by typing $3 \div 8 \times 2000$ into the calculator and press = (alternatively, we could type $\frac{3}{8} \times 2000$ and press =). The calculator then outputs the result, which is 750.

Next, we just have to add this amount to 2000. To do this we simply type + 2000 (which the calculator will treat as 'Ans + 2000') and press =. The calculator will then display the answer: 2750.

Example 2 has the same basic method, but it is a decrease rather than an increase, so in the second step we need to subtract rather than add.

We start by typing in $7 \div 12 \times 57$ and then pressing = (or $\frac{7}{12} \times 57$ and then =). This gives us $\frac{7}{12}$ of 57, which is 33.25.

Next we need to subtract 33.25 from 57. We can do this by type=ing in $57 - \text{Ans}$ and then pressing =. The calculator will subtract 33.25 from 57, giving us 23.75, which is the answer.

Given the fraction of itself that the initial amount was increased or decreased by, and the final amount, calculate the initial amount.

Examples

- 1) A number was increased by $\frac{3}{4}$ of itself to give 105. What was the original number?
- 2) A number was decreased by $\frac{2}{5}$ of itself to give 28.2. What was the original number?

Notes

To do this question, we have to start by working out what fraction of the original number the number we have been given is.

In Example 1, the original number has been increased by $\frac{3}{4}$ of itself to give 105. This means that 105 is $1\frac{3}{4}$ of the original number (because it was originally 1 of itself and then it gained $\frac{3}{4}$ of itself).

To get the original number we simply have to divide 105 by $1\frac{3}{4}$. We can do this on the calculator. Using the method explained previously to enter the mixed number, we type in $105 \div 1\frac{3}{4}$ and press =, which gives the answer: 60.

To understand why this method works, let's call the number we are looking for X. We know that $1\frac{3}{4}$ of X is 105. We can write this as an equation:

$$1\frac{3}{4} \times X = 105$$

If we divide both sides by $1\frac{3}{4}$, we get:

$$X = 105 \div 1\frac{3}{4}$$

If you prefer, you could first convert $1\frac{3}{4}$ to a top heavy fraction using the method explained previously. If you do this, you will find that it is equal to $\frac{7}{4}$. To get the answer, you then just divide 105 by $\frac{7}{4}$, which gives 60 (or you could do $105 \div 4 \times 7$).

In Example 2, the original number has been decreased by $\frac{2}{5}$ of itself to give 28.2.

First, we need to work out what fraction of the original number 28.2 is. If $\frac{2}{5}$ of the original amount has been lost, then what is left is $\frac{3}{5}$ of the original amount. If you are used to

working with fractions, you may be able to work this out in your head. If not, consider the fact that the original amount was 1 of itself and then it lost $\frac{2}{5}$ of itself. Therefore the fraction of it that is left is $1 - \frac{2}{5}$, which is $\frac{3}{5}$ (you can do this on the calculator).

Once we have worked out that 28.2 is $\frac{3}{5}$ of the original amount, we can use any of the methods explained previously to work out the original amount:

- $28.2 \div \frac{3}{5} = 47$
- $28.2 \div (3 \div 5) = 47$
- $28.2 \div 3 \times 5 = 47$